Network topology transition at criticality

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Many-body systems when continuous phase transition occurs are mainly built in the interrelationship between particles, implemented through many-body correlations. Some of them may exhibit so-called topological order hardly measured by experiments. Therefore we need, beyond mean-field theory, the complex-systems approach that stresses the systemic complexity of many-body network at criticality. According to our previous study [1], network space experiences the homogeneous-heterogeneous transition invisible in traditional phase transitions. The network robustness can be a useful indicator to capture the critical phenomena of phase transitions with/without symmetry breaking. In this work, we demonstrate the idea of the change of robust networks is successfully applied to the well-known 1D quantum and 2D classical XY models.

Determining the topological order of an interacting quantum system from its microscopic many-body entanglement is one of the recent goals of condensed matter theory. Traditional phases of matter and phase transitions are usually distinguished by local order parameters. Consider, for instance, a continuous phase transition, the critical point is accompanied by a diverging correlation length in Landau's symmetry breaking framework [2]. However, it becomes clear that some topologically ordered phases do not fall into this framework [3–5], such as fractional quantum Hall states [6], quantum spin liquids [7] and recently discovered topological insulators [8, 9]. Moreover, characterizing topological phase transitions between them is a rather difficult task due to the absence of local order parameters [10, 11]. There are recent advances in diagnosing the presence of topological order from knowledge of many-body ground states, including using entanglement entropy [12, 13], entanglement spectrum [14, 15], modular transformation [16], Chern number [17] and Z_2 topological invariant [18, 19]. They all have brought us closer to being able to tackle such important questions.

On the other hand, the complex network theory originating from graph theory has become one standard tool to analyze the structure and dynamics of real-world systems, which consists of overwhelming information [20–24]. The building blocks of a complex network include nodes (system's elements) and links (relation between two elements). Via the unique patterns of connections, the essential features of a collection of interacting elements can be unveiled by direct visualization and network analysis. Its application is prevailing in many fields, such as sociology, biology, informatics and many other interdisciplinary studies [25]. Thus the generic description is reasonably applicable to condensed matter systems.

An interesting question in the application of network analysis is: Can the critical phenomena in condensed matters be observed by complex network topology [26]? The answer could be tracked back to a common phenomenon of many real-world networks, network robustness, which is characterized by the connectivity of complex network. In light of extensive studies in various real-world networks [27–30], it will be interesting to explore the network robustness of many-body systems.

The focus of this work is to construct the weighted networks for the one-dimensional (1D) quantum and the two-dimensional (2D) classical XY models, respectively. The weight of network links we define carries quantum or classical correlations between particles in these two models. A significant observation is that the critical region in the 1D XY model can be distinguished by the novel network topology, in addition to typical entanglement entropy. We find that the entanglement entropy and the network robustness share similar mathematical structure but with different interpretations according to graph theory. The topology arising from the weighted networks is illustrated by the network robustness. It is also confirmed that a direct relationship between the change of the network robustness and the homogeneous-heterogeneous transition in weight distribution of network links [31]. Applying the same network property to the 2D XY model, we show that as well it may detect the Kosterlitz-Thouless (KT) transition point obtained by conventional quantities, e.g. spin stiffness.

Results

1D Quantum XY Model. The 1D quantum XY model consists of a chain of L spins with nearest neighbor interaction and transverse magnetic field [32]. There is an anisotropy parameter γ which is the ratio between x and y direction interaction, and a tunable transverse magnetic field strength λ identifying criticality in the Hamiltonian,

$$H_{XY} = -\frac{1}{2} \sum_{l} \left(\frac{1+\gamma}{2} \sigma_l^x \sigma_{l+1}^x + \frac{1-\gamma}{2} \sigma_l^y \sigma_{l+1}^y + \lambda \sigma_l z \right), \tag{1}$$

where l labels the spins and $\sigma_u^l (u = x, y, z)$ are the Pauli matrix. In the spin-$\frac{1}{2}$ chain, there are ferromagnetic-paramagnetic transitions. The phase transition is based on the transverse magnetic field and the anisotropy parameter. Let the anisotropic parameter $\gamma = 0$, we

recover the XX Hamiltonian with transverse magnetic field,

$$H_{XX} = -\frac{1}{2} \sum_l \left(\frac{1}{2} \sigma_l^x \sigma_{l+1}^x + \frac{1}{2} \sigma_l^y \sigma_{l+1}^y + \lambda \sigma_l z \right). \tag{2}$$

By applying Jordan-Wigner transformation to Eq.(2), we derive the Majorana fermionic Hamiltonian which is skew-symmetric [33, 34]. Further using Fourier transformation and Bogoliubov transformation, the ground state is expressed with the correlation matrix Γ_{mn}^A of the Majorana operator a_n

$$a_m a_n = \delta_{m,n} + i\Gamma_{mn}^A \tag{3}$$

where

$$\Gamma^A = \begin{pmatrix} \Pi_0 & \Pi_1 & \dots & \Pi_{L-1} \\ -\Pi_1 & \Pi_0 & & \vdots \\ \vdots & & \ddots & \vdots \\ -\Pi_{L-1} & \dots & \dots & \Pi_0 \end{pmatrix}, \tag{4}$$

$$\Pi_l = \begin{pmatrix} 0 & g_l \\ -g_l & 0 \end{pmatrix}, \tag{5}$$

with real coefficients g_l given, in the limit of an infinite chain, by

$$g_l = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{il\phi} \frac{cos\phi - \lambda - i\gamma sin\phi}{|cos\phi - \lambda - i\gamma sin\phi|}. \tag{6}$$

The adjacency matrix $\hat{A}_{ij}$ in the network representation is defined by the correlation matrix Γ^A. Note that the size of network space would be $2L$ owing to two Majorana fermions at each site. The weight of the network link is just the correlation matrix element between two nodes.

It is well-known that there exists the critical region $(0 < \lambda < 1)$ in the XX Hamiltonian with transverse magnetic field. As increasing λ, the paramagnetic phase would pass through the critical point $(\lambda=1)$ and change into the ferromagnetic phase. For the critical XX model, the entanglement entropy of the region $\Omega = \{1, L\}$ scales as

$$S = \frac{1}{3} \log_2 L + O(1). \tag{7}$$

The maximum entropy is reached when $\lambda = 0$. The entropy decreases while the transverse magnetic field increases until $\lambda = 1$ when the system goes into the ferromagnetic phase. Similar universal behavior of the model can be proven for Renyi entropy of order n,

$$S_n = \frac{1}{1-n} \log \left(Tr(\rho^n) \right), \tag{8}$$

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of which the reduced density matrix can be reconstructed from the correlation matrix Γ^A. On the other hand, the network robustness (defined in Section Methods) can be derived from

$$R = ln\Big(\frac{1}{N}\sum_{n=1}^{\infty}\frac{n!}{N_n}\Big) \tag{9}$$

where $N_n = Tr(\hat{A}^n)$ represents the strength of loops of length n in network space. At first glance, interestingly, the network robustness has the similar mathematical structure as the Renyi entropy when the adjacency matrix is defined as the reduced density matrix. The Renyi entropy of order n is just the n-th term of the sum in Eq.(8). In other words, the Renyi entropy of order n partly describes the robustness of abstract network space. We have to consider the Renyi entropy of all orders if the entire network space needs to be explored. Therefore, we reasonably speculate that the critical behavior of the XX model is more properly described by the network robustness.

In Fig.1, we calculate the network robustness of the critical XX model as a function of transverse magnetic field strength. Within the critical region ($0 < \lambda < 1$), the complex network system is rather fragile to random failures. But after being thrust into the ferromagnetic phase ($\lambda > 1$), the robustness of the network recovers. The interesting phenomenon can be understood by looking into the network space in details. Taking an example of $\lambda = 0.8$, the shape of the network whose nodes connect to each other with different link weights looks much like a "dimer liquid". Here the dimer we call is the strongest link highlighted in blue color in the inset of Fig.1. In fact, the dimer is formed by two Majorana fermions at the same site in real space. The other weak links highlighted in red color attempt to connect the dimers to form the "liquid-like" pattern. Above $\lambda = 1.0$, most of links between dimers vanish, and the network structure turns a "dimer liquid" into a "dimer gas". In the ferromagnetic region, all dimers become independent with respect to each other leading to the appearance of the dimer gas in the network space. It seems to involve some sort of "liquid-gas" transition in the network pattern. Owing to heterogeneous distribution of link weights (explained below), there exists the more robust network structure in the ferromagnetic phase. Furthermore, the critical point $\lambda = 1$ can be clearly found by the scenario as well.

In Fig.2, we analyze the homogeneous and heterogeneous network topologies in the 1D XY model. It shows in Fig.2(a) that the distance $r(= |i - j|)$ dependence of the adjacency

matrix element $\hat{A}_r$ changes as the phase transition occurs at $\lambda = 1$. In the critical region ($\lambda < 1$), the critical phase shows power-law behavior at all distances. Above the critical point ($\lambda > 1$), the ferromagnetic phase has only the non-zero matrix elements at $r = 1$ leading to the extremely heterogeneous weight distribution of network links. As shown in Fig.2(b), the evolution of the probability distribution $p(w)$ of weight w of network links can provide a clear picture to illustrate the change of network topologies. First, the weights of network links continuously distribute in the network space when $\lambda < 1$. As increasing λ, the weight distribution becomes more heterogeneous. Above $\lambda = 1$, it even comes to two delta functions at $w = 0$ and 1 leading to extreme heterogeneity. It is a well-known fact that complex weighted networks with the heterogeneous weight distribution of network links are much more robust to random failures or attacks [31, 35]. To put it another way, the homogeneous weight distribution is one of important features of fragile networks. Therefore, it can be expected that there is a homogeneous-heterogeneous network transition hidden in the quantum phase transition as changing network robustness. It may allow us to further define the network robustness as a detector in network space for identifying the quantum phase transition.

2D Classical XY Model. The other example is the 2D classical XY model in a square lattice of size N described by [36, 37]

$$-\sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j = \sum_{\langle i,j \rangle} \cos\left(\theta_i - \theta_j\right), \tag{10}$$

where θ_i is the angle of the 2D spin vector $\vec{S}_i$ at site i. In contrast to 1D quantum XY system, the true long-range order is completely washed out by thermal or quantum fluctuations and only its topology remains. In fact, its low-temperature phase forms a quasi-long-range order originating from the power-law spin correlation decay. There is a phase transition from this phase to the high-temperature disordered phase whose correlations decay exponentially with distances. Such a transition is known as the KT transition associated with the disappearance of the quasi-long-range order. Near the topological phase transition the system begins to lose spin stiffness that shows up a universal jump near the transition temperature $T_c = 0.9$. This approach has been often used to numerically extract T_c [38–40]. Other than the 1D quantum case, we define the adjacency matrix as the spin-spin correlation function:

$$\hat{A}_{ij} \equiv |\vec{S}_i \cdot \vec{S}_j|, \tag{11}$$

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which can be calculated by using the standard Monte Carlo simulation. In what follows, we will show that the network topology also enables the network robustness to illustrate the KT transition.

Figure 3 shows the temperature evolution of the network robustness in the 2D XY model. As mentioned above, the low-temperature topology gives rise to the power-law correlation decay, and hence links and nodes are homogeneously arranged in an irregular network pattern as illustrated in the inset. The network representation also looks like a "liquid" pattern. When temperature approaches T_c, the short-range spin correlations become more and more dominant. At much higher temperature ($T = 1.8$), each node has four strongest links to its neighbors as shown in the inset. The network topology is equivalent to a torus which corresponds to a square lattice with periodic boundary conditions. Now the network pattern becomes "crystal-like" in the network space. Hence, a "liquid-crystal" transition at T_c can be expected to happen in the network space. The reason behind the topology is that extremely short-range spin correlations of the high-temperature disordered phase indicate much larger link weights between the nearest-neighboring sites, and thus the network structure resembles a torus. Conventionally, spin stiffness has usually been used to determine the transition temperature. Here we demonstrate that the network robustness also have the ability to detect the KT transition.

Consider translational invariance, Figure 4(a) presents how the adjacency matrix element $\hat{A}_r$ changes as the KT transition occurs. For low temperature ($T = 0.7$), the quasi-long-range-ordered phase has the power-law correlation at large distances leading to the "liquid-like" network pattern. Not much far from the 1D example, Figure 4(b) shows that the weight distribution of network links looks log-normal. In other words, most links center their weights around $w = 0.75$ and display homogeneous distribution with a short tail ending at $w = 1$. The heterogeneity of the weight distribution appears as approaching the critical point ($T_c \sim 0.9$). Above the critical point ($T = 1.3$), the high-temperature phase instead shows that the spin-spin correlation is exponentially decaying with distances. The exponentially decaying correlation function in real space results in the "crystal-like" structure in network space. In the meantime, the peak of the distribution is moved to $w \sim 0$ with a long tail so that it becomes much more heterogeneous. Due to much broader weight distributions than the 1D case, it is rather difficult to come up with a clear order parameter in the 2D classical XY model. The same reasoning from the network topology can be applied to other

many-body systems without local order parameters. Even so, the network robustness can still be considered as a useful quantity to characterize the KT transition, in addition to traditional spin stiffness.

Conclusion. We have introduced the novel complex network analysis for helping us find how the network topology changes as a system undergoes the phase transition. In the 1D quantum XY model, the topology of the complex network is transited from "dimer liquid" to "dimer gas". At the critical point, a homogeneous-heterogeneous transition occurs in the weight distribution of network links. In contrast to entanglement entropy used in many literatures, the network robustness including more information about connectivity of the complex network may provide a more complete map to uncover the universal properties of the quantum phase transition. Thus the network robustness can be fairly considered as another detector in the quantum XY model.

In the 2D classical XY model, we have found that the critical phenomena can also be described by the change of the weighted network topology. The similar behavior that the structure of the weighted network varies from homogeneity to heterogeneity brings about the appearance of the network robustness to random failures. The robustness of the complex network is able to uncover a wealth of topological information underneath the classical spin-spin correlation, and further comprehend the mechanism of the phase transition without local order parameters. Notice that the complex network analysis is absolutely not designed for effectively calculating correlation functions but principally extracting the universal properties of the phase transitions without any symmetry breaking. The picture behind the weight distribution of network links can even provide significant information to comprehend the generic phase transitions no matter symmetry is broken or not. Therefore we propose that the network analysis approach presented in this work gives more general recipe for studying a variety of phase transitions in condensed matters.

Methods

In the language of complex network, each network of N nodes is described by its $N \times N$ adjacency matrix representation $\hat{A}$ [20]. A number of real systems, e.g. transportation networks, neural networks and so on, are better captured by the weighted network in which the link use weight to quantify their strength. In this work, we consider lattice sites as nodes

of the weighted network, with each weighted link between nodes i and j expressed by the element of the adjacency matrix $\hat{A}_{ij}$. The link carries the weight containing detailed information about the correlation between particles at different sites. In other words, the nodes (or say, lattice sites) are not connected by lattice bond but correlation between particles at corresponding sites so that a complete weighted network is formed.

The symmetric adjacency matrix $\hat{A}_{ij}$ in these two examples only has real entries. Hence the network gives rise to the signed structure whose link weights are allowed to be either positive or negative. A similar example is an acquaintance network in which we denote friendship by a positive link and animosity by a negative link [41]. Further definition of the two adjacency matrices corresponding to positive and negative correlations between lattice sites is possible. However, we find the results would not be changed if disregarding the sign of $\hat{A}_{ij}$, because the negative part is a tiny minority of the entire matrix. For simplicity, we will take the absolute value of $\hat{A}_{ij}$. Following the convention in the weighted complex network [42], we also take the absolute value of $\hat{A}_{ij}$ normalized by the maximum weight in the network $\frac{|A_{ij}|}{\max A_{ij}}$.

In complex network theory, a variety of network measures have been proposed to detect the structural robustness [43–46]. Natural connectivity derived mathematically from the graph spectrum has been introduced to measure the robustness of the weighted network structure [47–49]. More precisely, the network robustness can be described by the ability of a network to continue performing well after removing a fraction of nodes or links. One possible definition of the network robustness is similar to natural connectivity, given by

$$R = ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{\overline{\lambda}_i}\right) \tag{12}$$

where $\overline{\lambda}_i(\equiv \frac{\lambda_i}{\max \lambda_i})$ stands for the normalized eigenvalue of the adjacency matrix in order to avoid the enhancement of the network robustness as increasing the number of nodes N. Based on the definition of the network robustness, the reasoning behind the homogeneous-heterogeneous network transition is now clear. With the network transition from homogeneity to heterogeneity, exponentially decaying many-body correlation between nodes resulting far away from the critical point is the root cause of the robustness of complex network. The network robustness thus provides more information to comprehend traditional phase

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transitions than mean-field picture.

[1] Chou, C. P., Network Robustness: Detecting Topological Quantum Phases. *Sci. Rep.* **4**, 7526 (2014).

[2] Sachdev, S., *Quantum Phase Transitions* [33-34] (Cambridge University Press, 1999).

[3] Wen, X. G., *Quantum Field Theory of Many-Body Systems* [7-9] (Oxford University, New York, 2007).

[4] Nayak, C., Simon S. H., Stern A., Freedman, M. and Das Sarma S., Non-Abelian anyons and topological quantum computation. *Rev. Mod. Phys.* **80**, 1083 (2008).

[5] Bernevig, B. A., *Topological Insulators and Topological Superconductors* [196-200] (Princeton University Press, Princeton, 2013).

[6] Laughlin R. B., Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations. *Phys. Rev. Lett.* **50**, 1395 (1983).

[7] Anderson P. W., Resonating valence bonds: A new kind of insulator? *Mater. Res. Bul.* **8**, 153 (1973).

[8] Hasan M. Z. and Kane C. L., Colloquium: Topological insulators. *Rev. Mod. Phys.* **82**, 3045 (2010).

[9] Qi X.-L. and Zhang S. C., Topological insulators and superconductors. *Rev. Mod. Phys.* **83**, 1057 (2010).

[10] Wen X. G., Topological Order in Rigid States. *Int. J. Mod. Phys. B* **4**, 239 (1990).

[11] Carr L. D., *Understanding Quantum Phase Transitions* [169-174] (CRC Press, 2011).

[12] Kitaev A. and Preskill J., Topological Entanglement Entropy. *Phys. Rev. Lett.* **96**, 110404 (2006).

[13] Levin M. and Wen X. G., Detecting Topological Order in a Ground State Wave Function. *Phys. Rev. Lett.* **96**, 110405 (2006).

[14] Li H. and Haldane F. D. M., Entanglement Spectrum as a Generalization of Entanglement Entropy: Identification of Topological Order in Non-Abelian Fractional Quantum Hall Effect States. *Phys. Rev. Lett.* **101**, 010504 (2008).

[15] Qi X. L., Katsura H., and Ludwig A. W. W., General Relationship between the Entanglement Spectrum and the Edge State Spectrum of Topological Quantum States. *Phys. Rev. Lett.* **108**,

196402 (2012).

[16] Zhang Y., Grover T., Turner A., Oshikawa M., and Vishwanath A., Quasiparticle statistics and braiding from ground-state entanglement. *Phys. Rev. B* **85**, 235151 (2012).

[17] Thouless D. J., Kohmoto M., Nightingale M. P. and Nijs M. den , Quantized Hall Conductance in a Two-Dimensional Periodic Potential. *Phys. Rev. Lett.* **49**, 405 (1982).

[18] Kane C. L. and Mele E. J., Quantized Hall Conductance in a Two-Dimensional Periodic Potential. *Phys. Rev. Lett.* **95**, 146802 (2005).

[19] Kitaev, A., Unpaired Majorana fermions in quantum wires. *Phys. Usp.* **44**, 131 (2001).

[20] Albert R. and Barabasi A. L., Statistical mechanics of complex networks. *Rev. Mod. Phys.* **74**, 47 (2002).

[21] Newman, M. E. J., The structure and function of complex networks. *SIAM Rev.* **45**, 167 (2003).

[22] Dorogovtsev S. N. and Mendes J. F. F., Evolution of networks. *Adv. Phys.* **51**, 1079 (2002).

[23] Boccaletti S., Latora V., Moreno Y., Chavez M. and Hwang D. U., Complex networks: Structure and dynamics. *Phys. Rep.* **424**, 175 (2006).

[24] Dorogovtsev S. N., Goltsev A. V. and Mendes, J. F. F., Critical phenomena in complex networks. *Rev. Mod. Phys.* **80**, 1275 (2008).

[25] Sethna J. P., *Statistical Mechanics: Entropy, Order Parameters and Complexity* [25-26] (Oxford University Press, 2006).

[26] Chou C. P. and Chang M. C., Network topology: Detecting topological phase transitions in Kitaev chain and plane rotator. arXiv:1308.0255 (2013).

[27] Albert R., Jeong H. and Barabasi A.-L., Internet: Diameter of the World-Wide Web. *Nature* **401**, 130 (1999).

[28] Newman M. E. J., The structure of scientific collaboration networks. *Proc. Natl. Acad. Sci.* **98**, 404 (2001).

[29] Dodds P. S., Muhamad R. and Watts J. D., An Experimental Study of Search in Global Social Networks. *Science* **301**, 827 (2003).

[30] Backstorm L., Boldi P., Rosa M., Ugander J. and Vigna S., *Proceeding of the fourth Annual ACM Web Science Conference(WebSci)*, [33-42] (2012).

[31] Albert R., Jeong H. and Barabasi A. L., Error and attack tolerance of complex networks. *Nature* **406**, 378 (2000).

[32] Latorre J. I., Rico E. and Vidal G., Ground state entanglement in quantum spin chains. arXiv:quant-ph/0304098 (2003).

[33] Kitaev A., Fault-tolerant quantum computation by anyons. *Ann. Phys.* **303**, 2 (2003).

[34] Levin M. A. and Wen X. G., String-net condensation: A physical mechanism for topological phases. *Phys. Rev. B* **71**,045110(2005).

[35] Wang B., Tang H., Guo C. and Xiu Z., Entropy optimization of scale-free networks' robustness to random failures. *Physica A* **363**, 591 (2006).

[36] Kosterlitz J. M. and Thouless D. J., Ordering, metastability and phase transitions in two-dimensional systems. *J. Phys. C* **6**, 1181 (1973).

[37] Kosterlitz J. M., The critical properties of the two-dimensional xy model. *J. Phys. C* **7**, 1046 (1974).

[38] Nelson D. R. and Kosterlitz J. M., Universal Jump in the Superfluid Density of Two-Dimensional Superfluids. *Phys. Rev. Lett.* **39**, 1201 (1977).

[39] Weber H. and Minnhagen P., Monte Carlo determination of the critical temperature for the two-dimensional XY model. *Phys. Rev. B* **37**, 5986 (1988).

[40] Hasenbusch M., The Binder Cumulant at the Kosterlitz-Thouless Transition. *J. Stat. Mech.* P08003 (2008).

[41] Facchetti G.,Iacono G. and Altafini1 C., Computing global structural balance in large-scale signed social networks. *Proc. Natl. Acad. Sci.* **108**, 20953 (2011).

[42] Saramaki J., Kivela M., Onnela J. P., Kaski K. and Kertesz J., Generalizations of the clustering coefficient to weighted complex networks. *Phys. Rev. E* **75**, 027105 (2007).

[43] Harary F., Conditional connectivity. *Networks* **13**, 347 (1983).

[44] Krishnamoorthy M. S. and Krishnamurthy B., Fault diameter of interconnection networks. *Comput. Math. Appl.* **13**, 577 (1987).

[45] Esfahanian A. H. and Hakimi S. L., On computing a conditional edge connectivity of a graph. *J. Inf. Process. Lett.* **27**, 195 (1988).

[46] Bauer G. and Bolch G., Analytical approach to discrete optimization of queuing-networks. *Comput. Commun.* **13**, 494 (1990).

[47] Wu J., Barahona M., Tan Y. J. and Deng H. Z., Natural connectivity of complex networks. *Chin. Phys. Lett.* **27**, 078902 (2010).

[48] Wu J., Barahona M., Tan Y. J. and Deng H. Z., Robustness of random graphs based on graph

spectra. *Chaos* **22**, 043101 (2012).

[49] Estrada E., Hatano N. and Benzi, M., The physics of communicability in complex networks. *Phys. Rep.* **514**, 89 (2012).

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Competing Financial Interests:

The authors declare no competing financial interests.

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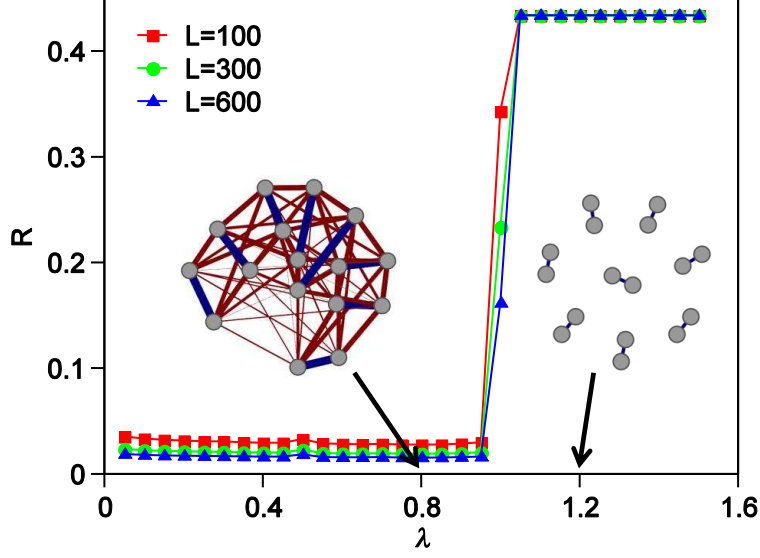Correspondence and requests for materials should be addressed to cpc63078@gmail.com.

FIG. 1: The network robustness R of the 1D quantum XY model vs transverse magnetic field strength λ for different chain length L. Network representations for λ below and above the critical point are shown in the inset.
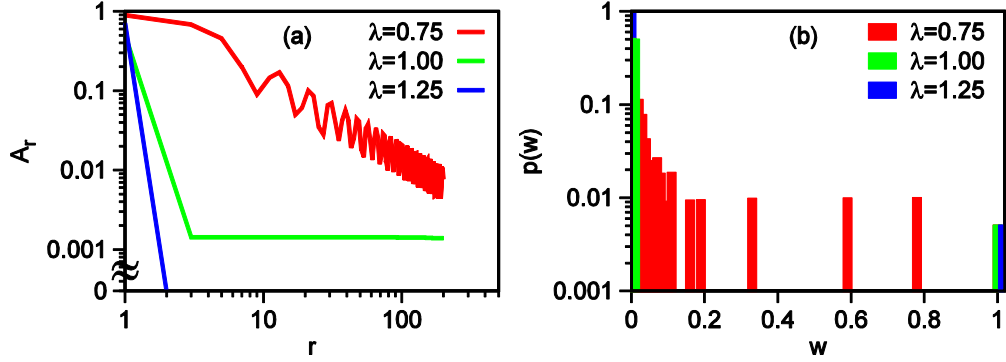

FIG. 2: (a) The adjacency matrix element A_r as a function of distance r and (b) The probability distribution $p(w)$ of weight w of network links for different transverse magnetic field strength λ. The bin size is chosen for clear demonstrations. The chain length $L=200$.
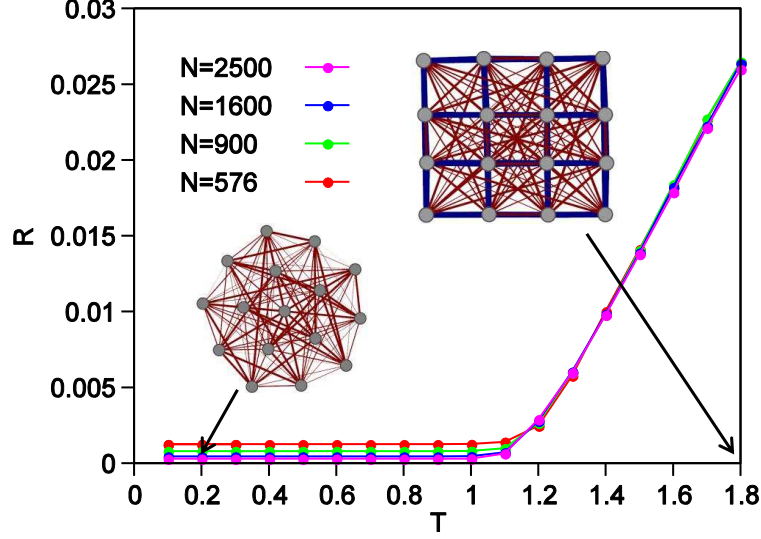
FIG. 3: The network robustness R of the 2D classical XY model vs temperature T for different lattice size N. Network representations for T below and above the critical point are shown in the inset.
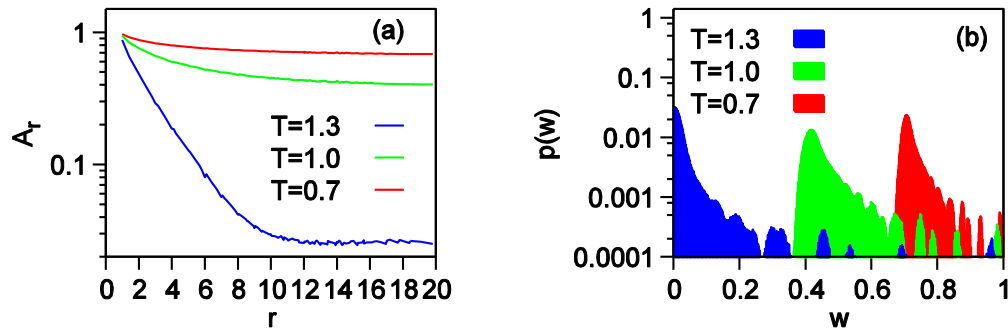

FIG. 4: (a) The adjacency matrix element A_r as a function of distance r and (b) The probability distribution $p(w)$ of weight w of network links for different temperature T. The lattice size N=900.